5-6 Videos Guide

5-6a

- Description of and notation for parametric surfaces
  - $\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$
  - Parametric equations are $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$
- Planes as parametric surfaces
  - A plane that contains a point P_0, which corresponds to $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$, and vectors $\mathbf{a}$ and $\mathbf{b}$ has equation $\mathbf{r}(u, v) = \mathbf{r}_0 + u\mathbf{a} + v\mathbf{b}$
- Spheres as parametric surfaces
  - A sphere of radius a and center $(0, 0, 0)$ is parameterized as $\mathbf{r}(\phi, \theta) = a\cos\theta\sin\phi\ \mathbf{i} + a\sin\theta\sin\theta\ \mathbf{j} + a\cos\phi\ \mathbf{k}$, $0 \le \phi \le \pi$, $0 \le \theta \le 2\pi$
- Functions as parametric surfaces
  - A surface given by a function $z = f(x, y)$ is parametrized as $\mathbf{r}(x, y) = x\,\mathbf{i} + y\,\mathbf{j} + f(x, y)\,\mathbf{k}$

Exercises:

5-6b

- Identify the surface with the given vector equation.
  - $\mathbf{r}(u, v) = u^2\ \mathbf{i} + u\cos v\ \mathbf{j} + u\sin v\ \mathbf{k}$
  - $\mathbf{r}(s, t) = \langle 3\cos t, s, \sin t \rangle$, $-1 \le s \le 1$

5-6c

- Find a parametric representation for the surface.
  - The part of the part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$.
  - The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

5-6d

- Tangent planes
  - The tangent plane to a surface given by $\mathbf{r}(u, v)$ has normal vector $\mathbf{r}_u \times \mathbf{r}_v$

Exercise:

- Find an equation of the tangent plane to the given parametric surface at the specified point.
  $x = u^2 + 1,\ y = v^3 + 1,\ z = u + v;\ (5, 2, 3)$

5-6e

- Surface area
  - If a smooth parametric surface S is given by $\mathbf{r}(u, v)$, $(u, v) \in D$ and S is covered exactly once as (u, v) covers D, then $A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v|\, dA$

- If $z = f(x, y)$, then $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{1 + (f_x)^2 + (f_y)^2}$
- For a surface of revolution obtained by rotating $f(x)$, $a \leq x \leq b$ about the x-axis, use $x = x$, $y = f(x) \cos\theta$, $z = f(x) \sin\theta$, $0 \leq \theta \leq 2\pi$, so that $A(S) = \int_a^b \int_0^{2\pi} |\mathbf{r}_x \times \mathbf{r}_\theta| \, dA$

5-6f

Exercise:

- Find the area of the part of the surface $z = 4 - 2x^2 + y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.